Research Proposal: uniformizing the study of the logical key properties

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Safe Programs is a growing need since they are part of critical systems like airplanes. Different approaches are developed for this purpose, as model checking for instance.
Proof theory has multiple application in this field. Logic and Type theory gave rise to proof assistant, used to prove correctness of programs. Automatic proof search is also widely used in this field.

The aim of my visit to Pr. Okada is to study the properties of such systems.

Linear Logic is a theoretical framework introduced in the mid 80ies.

- Has been proven very relevant to model resources.
- Expresses very precisely the meaning of other logical frames.
- Okada's Laboratory is one of the specialists on those topics.

- Normalization is stronger than semantical proofs.
- Semantical proofs are more acute.
- In particular, a semantical proof describes a proof search algorithm.
- Mitsu Okada has studiyed the links between the two methods in Linear Logic.
- Some other scientists (Dowek, Coquand) began the same work in Deduction Modulo and other frames.
- But the relation is still not clear.

The Curry-Howard isomorphism is a correspondence between proofs, data types and programs:

- A proof of $A \Rightarrow B$ is a program associating a proof of B to any proof of A.
- A proof of $\forall x\ A(x)$ is a program associating a proof of A(t) to any term t.

This is the basis for proof assitants like Coq, Isabelle or Agda.

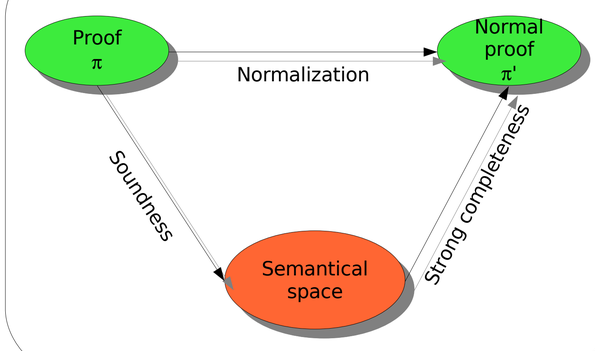
The key property of all of these systems: cut elimination.

- It is the ability to transform a proof in a normal form.
- Through the Curry-Howard isomorphism it is equivalent to the termination of a program.
- It implies many other properties, as decidability, analiticity, and allows effective implantation.

- One of the main aims of my visit in Japan is to study the relations between both methods.
- I will also study the links between all the different semantical frames, that seems crucial to understand those links.
- A comparison with some other abstract approach by Terui can be useful.
- It also can result to a combination of Deduction Modulo with Linear Logic.

Deduction Modulo is a powerful way to combine deduction and computation:
- It replaces elementary deduction steps by a sequence of rewriting: 2 + 2 computes to 4.
- Is therefore well fitted for automated theorem proving.
- Allows a great flexibility.
- It is harder to prove the good properties (they may fail). But they are expressed in an uniform way.
- Is mainly studied at INRIA, France.

Two methods for cut elimination:



Expected outcomes of the staying:

- The work I will start here will hopefully turn into a long-term collaboration between both labs.
- I will also bring my knowledge of Coq to express and formally prove some results studied in Okada's Laboratory.