A Modular Static Analysis Approach to Affine Loop Invariants Detection

Corinne Ancourt, Fabien Coelho and Francois Irigoin

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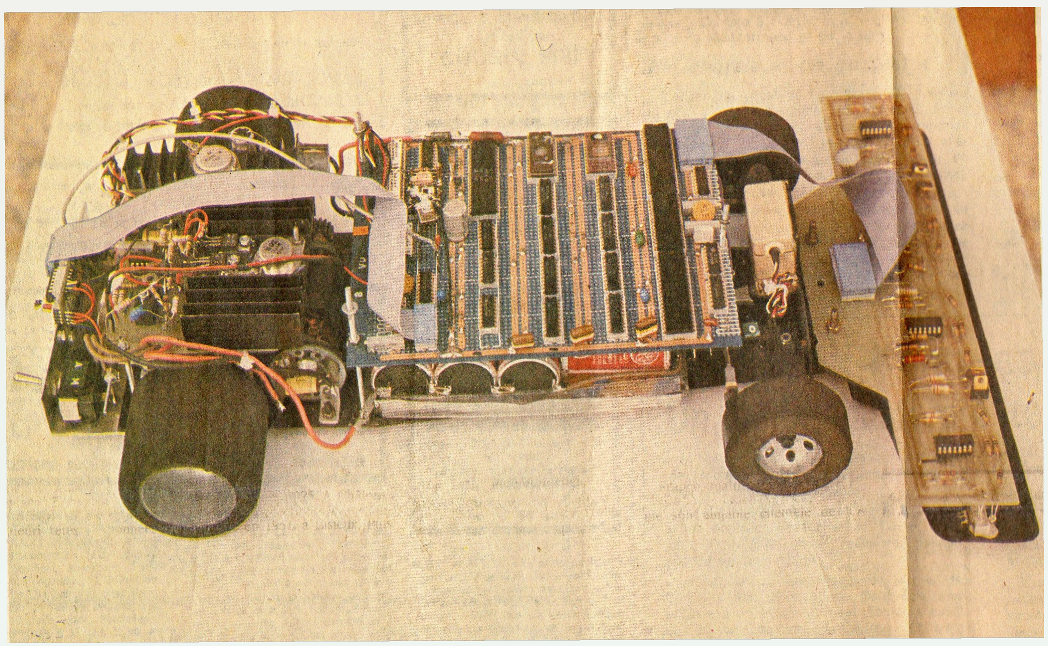
Car racing ....

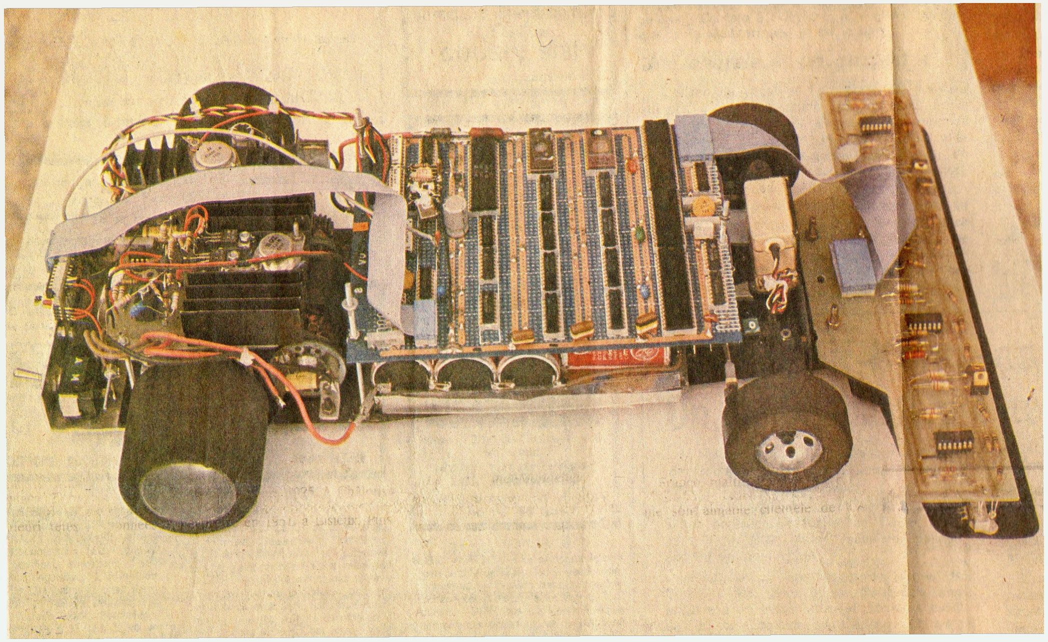
I.1.1

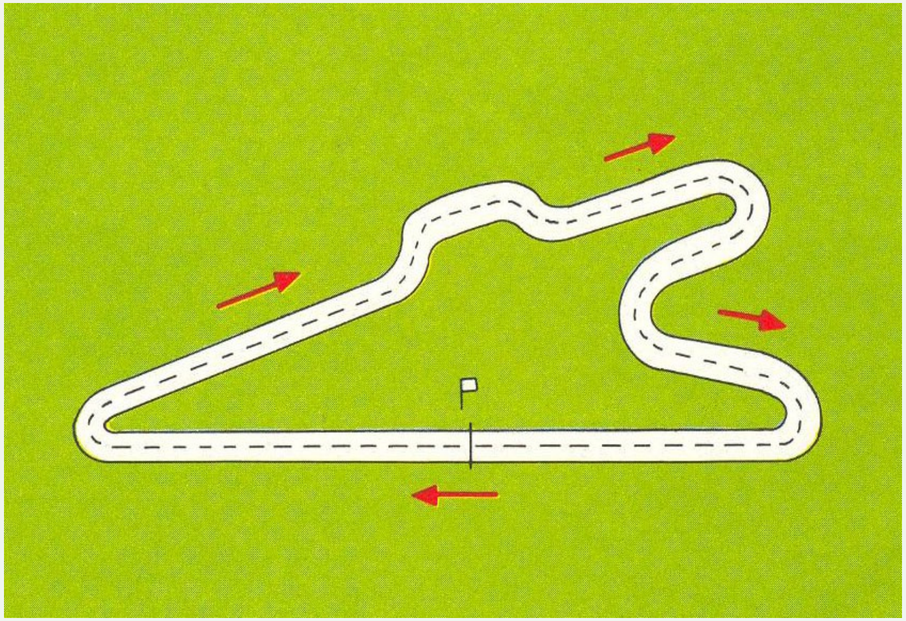
Intelligent Car - 1980



Car racing ....

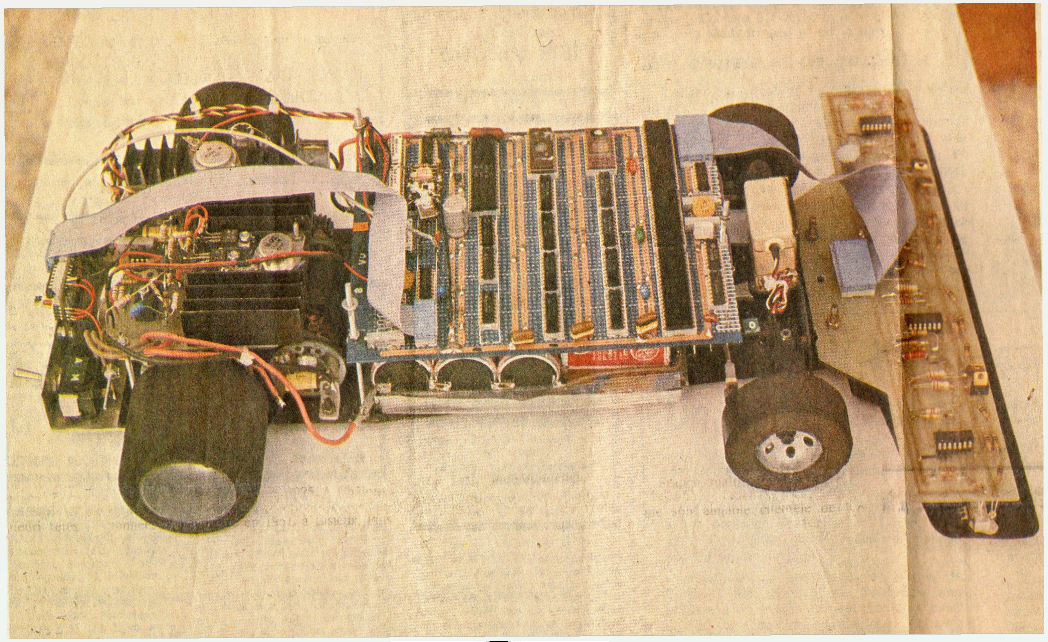
Intelligent Car - 1980

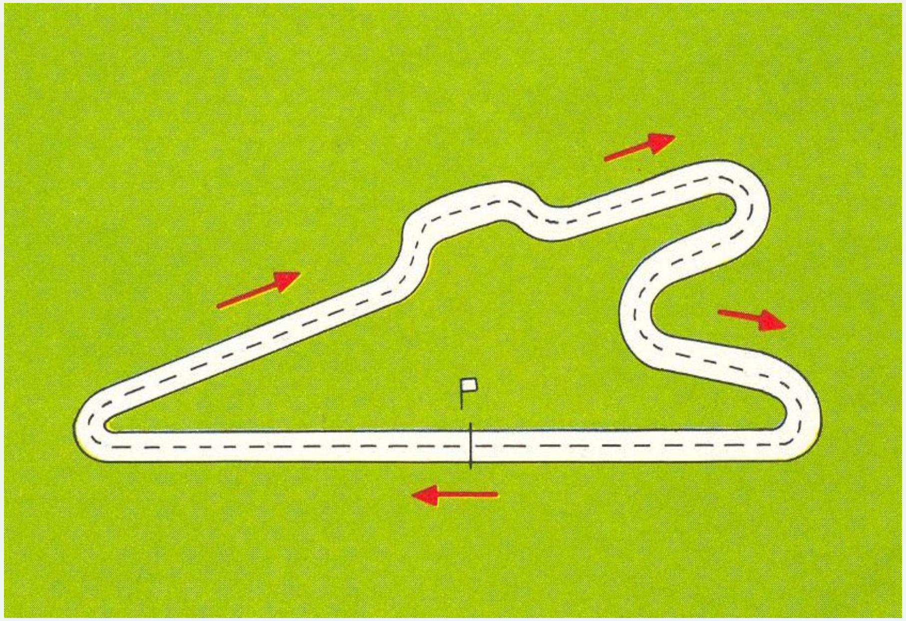


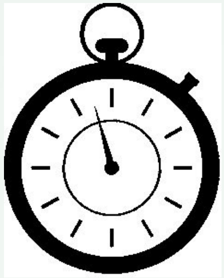


Car racing ....

Intelligent Car - 1980

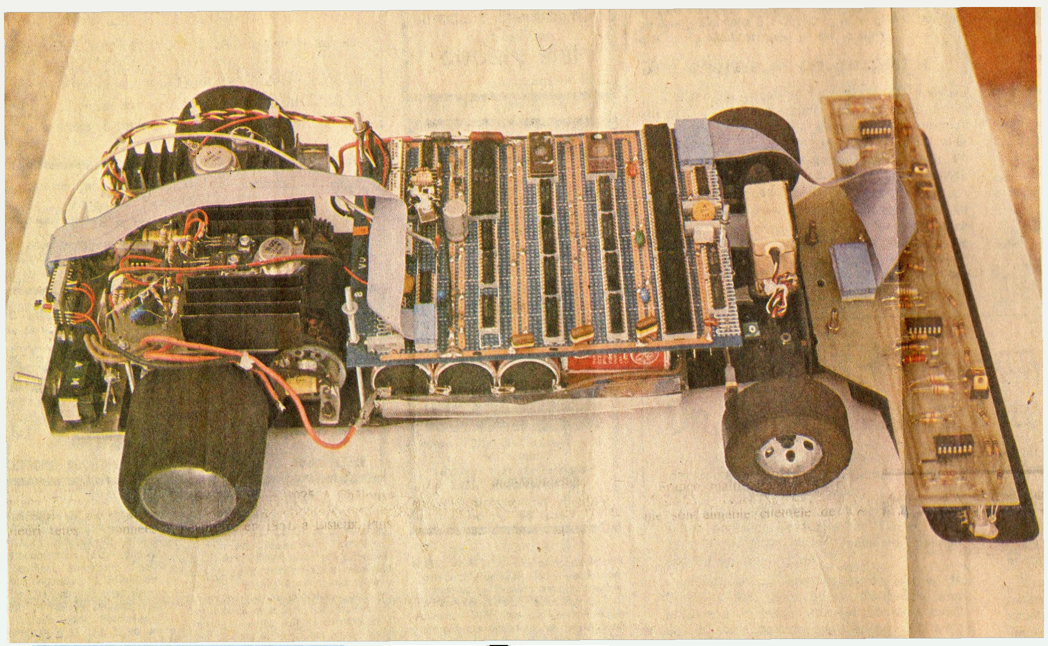


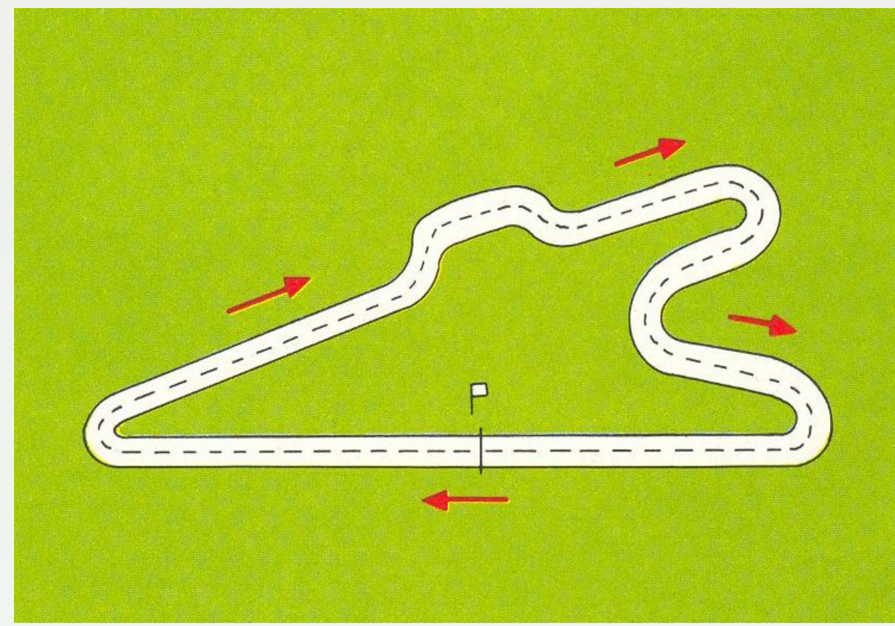


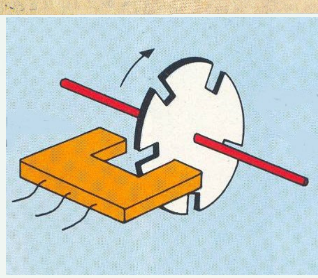


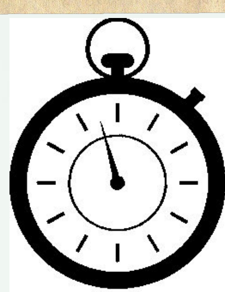
Car racing ....

Intelligent Car - 1980





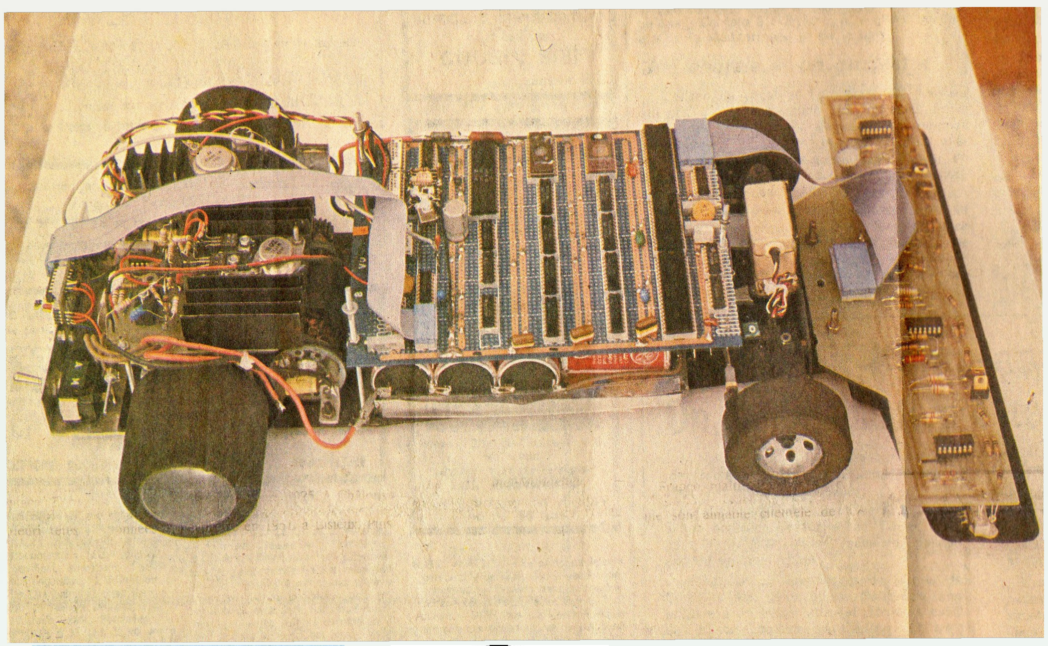


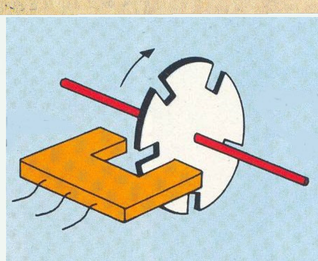


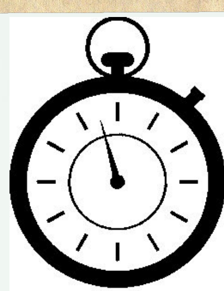
Car racing ....

I.1.1

Intelligent Car - 1980







Robot Car safety – Halbwachs 93 extended

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```

Scientific Computing

I.2.1

```
      SUBROUTINE COSTI (N,WSAVE)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DIMENSION  WSAVE(*)
      PARAMETER (PI = 3.14159265358979)
      IF (N .LE. 3) THEN
      RETURN
      ENDIF
      NM1 = N-1
      NP1 = N+1
      NS2 = N/2
      DT = PI/FLOAT(NM1)
      KC = NP1-1
      DO 101 K=2,NS2
        KC = KC-1
        FK = FLOAT(K-1)
        WSAVE(K) = 2.D0*DSIN(FK*DT)
        WSAVE(KC) = 2.D0*DCOS(FK*DT)
  101 CONTINUE
C     CALL RFFTI (NM1,WSAVE(N+1))
      RETURN
      END
```

Computing Invariants for

- **Optimization**
- **Dependence analysis**
- **Parallelization**
- **Array access bound checking**
- **Array access initialization checking**
- **Property verification**
- **...**

Scientific Computing

I.2.1

```
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        FK = FLOAT(K-1)
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        WSAVE(KC) = 2.D0*DCOS(FK*DT)
  101 CONTINUE
C       CALL RFFTI (NM1,WSAVE(N+1))
      RETURN
      END
```

```
C  P(KC,NM1,NP1,NS2){KC==N, KC==NM1+1,
C  KC==NP1-1, 4<=KC, 2NS2<=KC, KC<=2NS2+1}

      DO 101 K = 2, NS2

C  P(K,KC,NM1,NP1,NS2){K+KC==N+2, N==NM1+1,
C  N==NP1-1, K<=NS2, KC<=N, 4<=N, 2NS2<=N,
N<=2NS2+1}

        KC = KC-1

C  P(K,KC,NM1,NP1,NS2){K+KC==N+1, N==NM1+1,
C  N==NP1-1, K<=NS2, KC+1<=N, 4<=N, 2NS2<=N,
N<=2NS2+1}

        FK = FLOAT(K-1)
        WSAVE(K) = 2.D0*DSIN(FK*DT)
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101       CONTINUE
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Scientific Computing

I.2.1

```
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      NM1 = N-1
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C  P(KC,NM1,NP1,NS2){KC==N, KC==NM1+1,
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    DO 101 K = 2, NS2
      KC = KC-1

C  P(K,KC,NM1,NP1,NS2){K+KC==N+1, N==NM1+1,
C  N==NP1-1, K<=NS2, KC+1<=N, 4<=N, 2NS2<=N,
N<=2NS2+1}

      FK = FLOAT(K-1)
      WSAVE(K) = 2.D0*DSIN(FK*DT)
      WSAVE(KC) = 2.D0*DCOS(FK*DT)
101     CONTINUE
```

Scientific Computing

I.2.1

```
      SUBROUTINE COSTI (N,WSAVE)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
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      PARAMETER (PI = 3.14159265358979)
      IF (N .LE. 3) THEN
      RETURN
      ENDIF
      NM1 = N-1
      NP1 = N+1
      NS2 = N/2
      DT = PI/FLOAT(NM1)
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      DO 101 K=2,NS2
        KC = KC-1
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```
C  P(KC,NM1,NP1,NS2){KC==N, KC==NM1+1,
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      DO 101 K = 2, NS2
        KC = KC-1

C  P(K,KC,NM1,NP1,NS2){K+KC==N+1, N==NM1+1,
C  N==NP1-1, K<=NS2, KC+1<=N, 4<=N, 2NS2<=N,
N<=2NS2+1}
```

Dependence Test for WSAVE:
K == KC , K+KC==N+1
2K == N+1, K<=NS2,
N+1 <= 2NS2 , 2NS2<=N
N+1 <= N

```
        FK = FLOAT(K-1)
        WSAVE(K) = 2.D0*DSIN(FK*DT)
        WSAVE(KC) = 2.D0*DCOS(FK*DT)
101     CONTINUE
```

Modularity: Preconditions and Transformers

I.3.1

```
int main()
{
  float a[10][10], b[10][10], h;
  int i, j;

  for(i = 1; i <= 10; i += 1)

    for(j = 1; j <= 10; j += 1)

      b[i][j] = 1.0;

  h = 2.0;

  func1(10, 10, a, b, h);

  for(i = 1; i <= 10; i += 1)

    for(j = 1; j <= 10; j += 1)

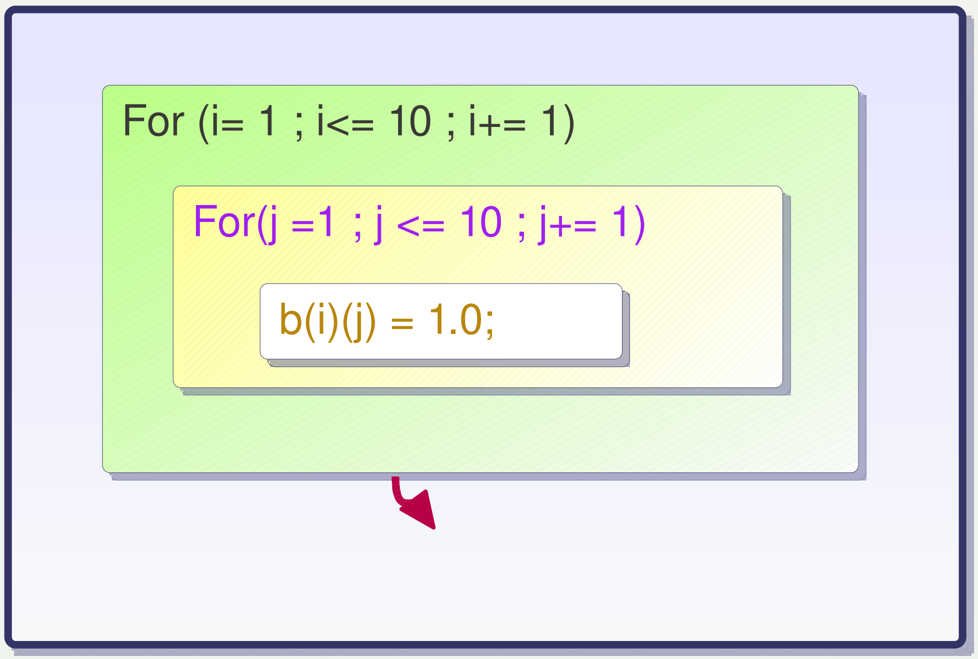
      fprintf(stderr, "a[%d] = %f \n", i, a[i][j]);
}
```

Modularity: Preconditions and Transformers

I.3.1

```
// P() {}
int main()
{
   float a[10][10], b[10][10], h;
   int i, j;
// P() {}
   for(i = 1; i <= 10; i += 1)
// P(i,j) {1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
// P(i,j) {1<=i, i<=10, 1<=j, j<=10}
         b[i][j] = 1.0;
// P(i,j) {i==11, j==11}
   h = 2.0;
// P(h,i,j) {2.0==h, i==11, j==11}
   func1(10, 10, a, b, h);
// P(h,i,j) {2.0==h, i==11, j==11}
   for(i = 1; i <= 10; i += 1)
// P(h,i,j) {2.0==h, 1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
// P(h,i,j) {2.0==h, 1<=i, i<=10, 1<=j, j<=10}
         fprintf(stderr, "a[%d] = %f \n", i, a[i][j]);
}
```

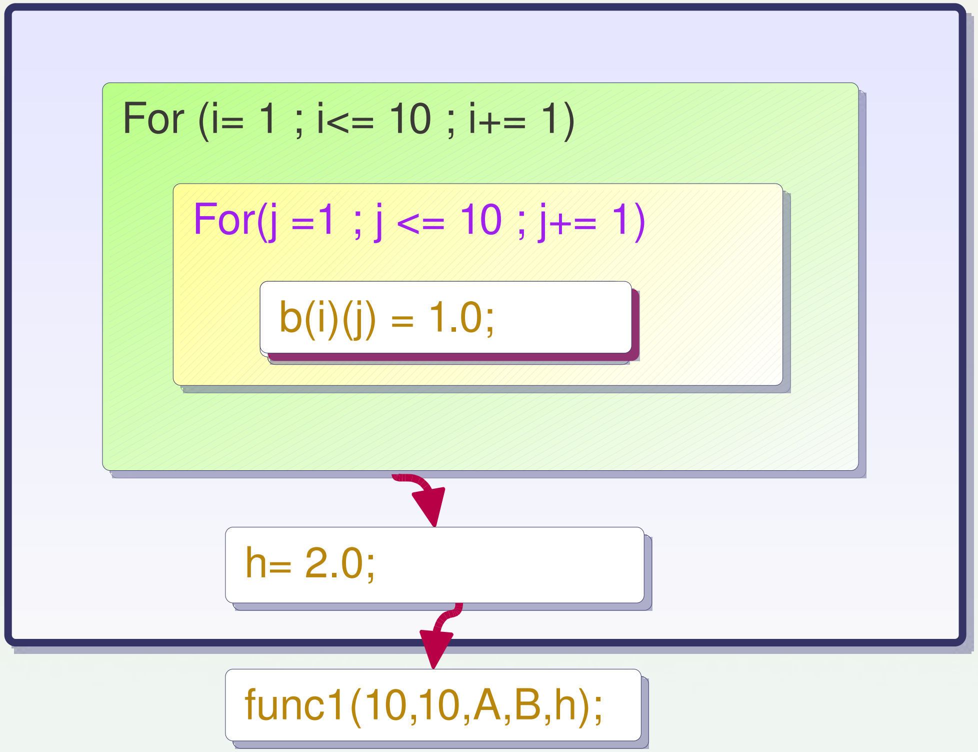
Modularity: Preconditions and Transformers




```
// P() {}
int main()
{
   float a[10][10], b[10][10], h;
   int i, j;
// P() {}
   for(i = 1; i <= 10; i += 1)
// P(i,j) {1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
// P(i,j) {1<=i, i<=10, 1<=j, j<=10}
         b[i][j] = 1.0;
// P(i,j) {i==11, j==11}
   h = 2.0;
// P(h,i,j) {2.0==h, i==11, j==11}
   func1(10, 10, a, b, h);
// P(h,i,j) {2.0==h, i==11, j==11}
   for(i = 1; i <= 10; i += 1)
// P(h,i,j) {2.0==h, 1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
// P(h,i,j) {2.0==h, 1<=i, i<=10, 1<=j, j<=10}
         fprintf(stderr, "a[%d] = %f \n", i, a[i][j]);
}
```

Modularity: Preconditions and Transformers

I.3.1

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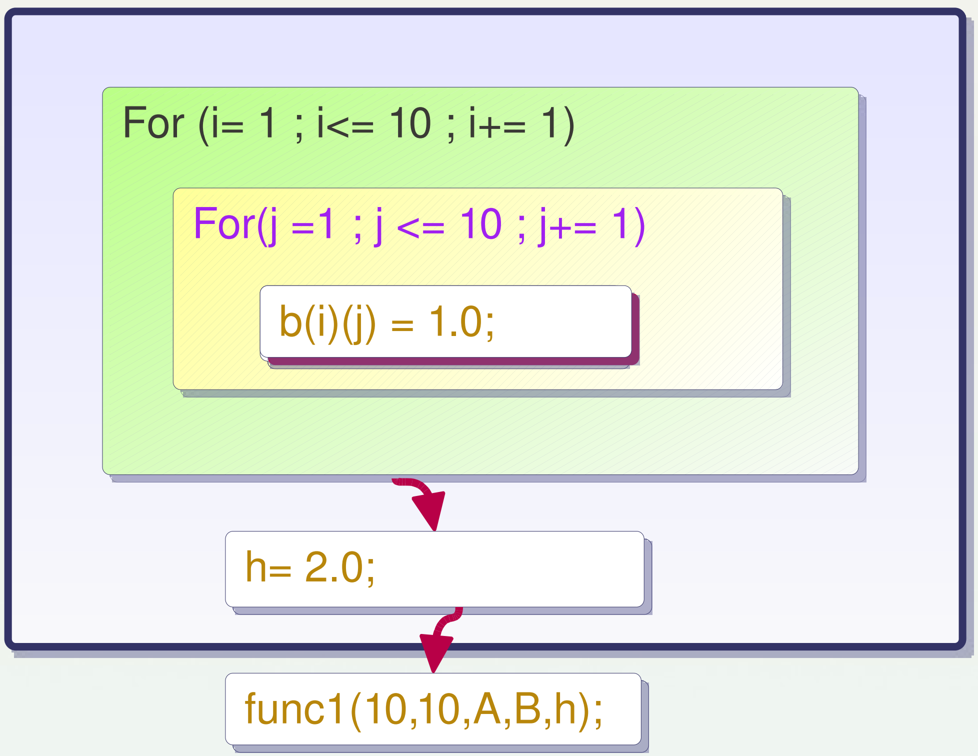
//  P() {}
int main()
{
   float a[10][10], b[10][10], h;
   int i, j;
//  P() {}
   for(i = 1; i <= 10; i += 1)
//  P(i,j) {1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
//  P(i,j) {1<=i, i<=10, 1<=j, j<=10}
         b[i][j] = 1.0;
//  P(i,j) {i==11, j==11}
   h = 2.0;
//  P(h,i,j) {2.0==h, i==11, j==11}
   func1(10, 10, a, b, h);
//  P(h,i,j) {2.0==h, i==11, j==11}
   for(i = 1; i <= 10; i += 1)
//  P(h,i,j) {2.0==h, 1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
//  P(h,i,j) {2.0==h, 1<=i, i<=10, 1<=j, j<=10}
         fprintf(stderr, "a[%d] = %f \n", i, a[i][j]);
}
```

Modularity: Preconditions and Transformers




```
// P() {}
int main()
{
   float a[10][10], b[10][10], h;
   int i, j;
// P() {}
   for(i = 1; i <= 10; i += 1)
// P(i,j) {1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
// P(i,j) {1<=i, i<=10, 1<=j, j<=10}
         b[i][j] = 1.0;
// P(i,j) {i==11, j==11}
   h = 2.0;
// P(h,i,j) {2.0==h, i==11, j==11}
   func1(10, 10, a, b, h);
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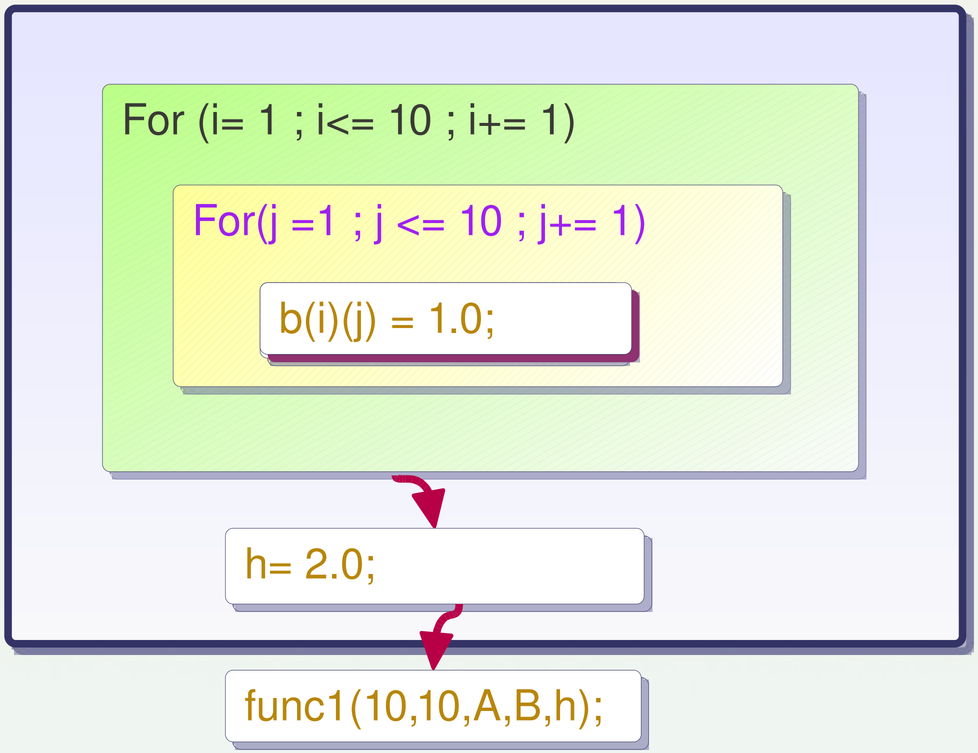
// P(h,i,j) {2.0==h, 1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
// P(h,i,j) {2.0==h, 1<=i, i<=10, 1<=j, j<=10}
         fprintf(stderr, "a[%d] = %f \n", i, a[i][j]);
}
```

Modularity: Preconditions and Transformers

I.3.1




```
//  P() {}
int main()
{
   float a[10][10], b[10][10], h;
   int i, j;
//  P() {}
   for(i = 1; i <= 10; i += 1)
//  P(i,j) {1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
//  P(i,j) {1<=i, i<=10, 1<=j, j<=10}
         b[i][j] = 1.0;
//  P(i,j) {i==11, j==11}
   h = 2.0;
//  P(h,i,j) {2.0==h, i==11, j==11}
   func1(10, 10, a, b, h);
//  P(h,i,j) {2.0==h, i==11, j==11}
   for(i = 1; i <= 10; i += 1)
//  P(h,i,j) {2.0==h, 1<=i, i<=10}
      for(j = 1; j <= 10; j += 1)
//  P(h,i,j) {2.0==h, 1<=i, i<=10, 1<=j, j<=10}
         fprintf(stderr, "a[%d] = %f \n", i, a[i][j]);
}
```

Principle: Each Function is Analyzed Once
Summaries must be built

Affine Transitive Closure Algorithm

II.0.1

Affine Transitive Closure Algorithm

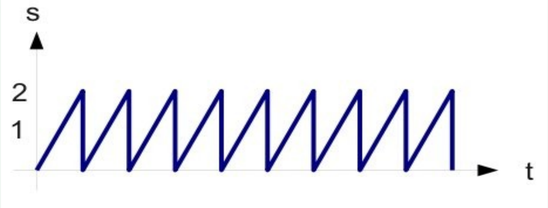
Car safety

II.1.1

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```



Car safety

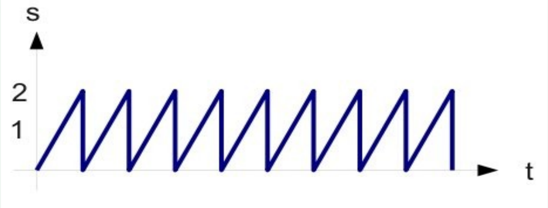
II.1.1

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```

T(d,s) {d==d'+1, s==s'+1, 0<=n, t<=n, s<=3}



Bottom-up propagation of Transformers

Car safety

II.1.1

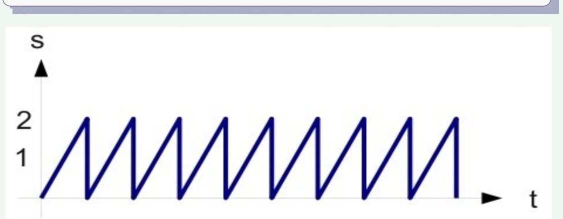
```
void voiture05(int n)
{
 int s = 0, t = 0, d = 0;

 assert(n>=0);
 while(s <= 2 && t <= n) {
  if(alea() > 0.)
    t++, s = 0;
  else
    d++, s++;
 }

 if(d <= 2*n+3)
  printf("safe");
 else
  printf("crashed!");
}
```

T(s,t) {s==0, t==t'+1, 0<=n, t<=n+1, s'<=2}

T(d,s) {d==d'+1, s==s'+1, 0<=n, t<=n, s<=3}



Bottom-up propagation of Transformers

Car safety

II.1.1

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

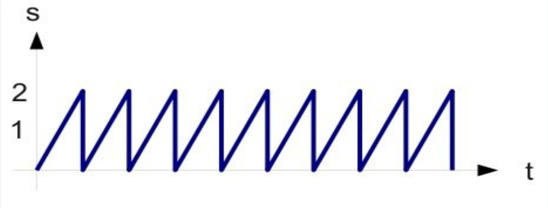
  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```

T(d,s,t) {d+t==d'+t'+1, 0<=n, t'<=n, s'+3t'+1<=s+3t, s+3t<=3t'+3, t'<=t, t<=t'+1}

T(s,t) {s==0, t==t'+1, 0<=n, t<=n+1, s'<=2}

T(d,s) {d==d'+1, s==s'+1, 0<=n, t<=n, s<=3}



Bottom-up propagation of Transformers

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```
void voiture05(int n)
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  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

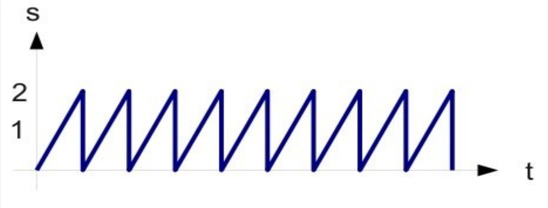
  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```

T(d,s,t) {d'==0, s'==0, t'==0, 0<=d, s<=3d, s<=2d+2t, d<=s+2t, 0<=n, t<=n, s<=2, 0<=t}

T(d,s,t) {d+t==d'+t'+1, 0<=n, t'<=n, s'+3t'+1<=s+3t, s+3t<=3t'+3, t'<=t, t<=t'+1}

T(s,t) {s==0, t==t'+1, 0<=n, t<=n+1, s'<=2}

T(d,s) {d==d'+1, s==s'+1, 0<=n, t<=n, s<=3}



Bottom-up propagation of Transformers

Car safety

II.1.1

Top-down propagation of Preconditions

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

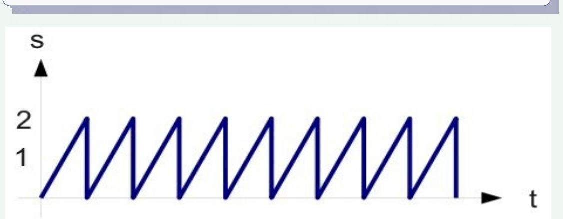
  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```

P(d,s,t) {d==0, s==0, t==0, 0<=n}

T(d,s,t) {d+t==d'+t'+1, 0<=n, t'<=n, s'+3t'+1<=s+3t, s+3t<=3t'+3, t'<=t, t<=t'+1}

T(s,t) {s==0, t==t'+1, 0<=n, t<=n+1, s'<=2}

T(d,s) {d==d'+1, s==s'+1, 0<=n, t<=n, s<=3}



Bottom-up propagation of Transformers

Car safety

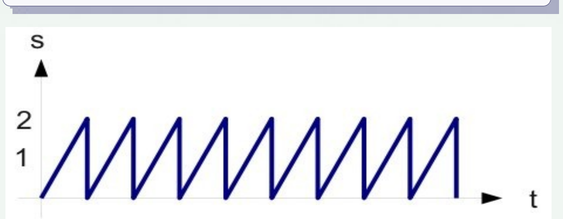
II.1.1

Top-down propagation of Preconditions

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```



P(d,s,t) {d==0, s==0, t==0, 0<=n}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, **t<=n,** **s<=2,** 0<=t}

T(s,t) {s==0, t==t'+1, 0<=n, t<=n+1, s'<=2}

T(d,s) {d==d'+1, s==s'+1, 0<=n, t<=n, s<=3}

Bottom-up propagation of Transformers

Car safety

II.1.1

Top-down propagation of Preconditions

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

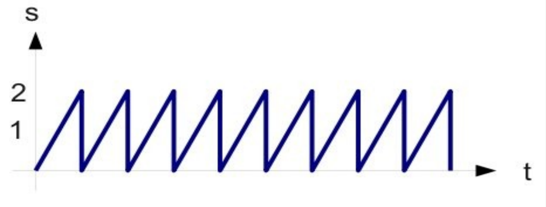
  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```

P(d,s,t) {d==0, s==0, t==0, 0<=n}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, **t<=n, s<=2,** 0<=t}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, t<=n, s<=2, 0<=t}

T(d,s) {d==d'+1, s==s'+1, 0<=n, t<=n, s<=3}



Bottom-up propagation of Transformers

Car safety

Top-down propagation of Preconditions

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

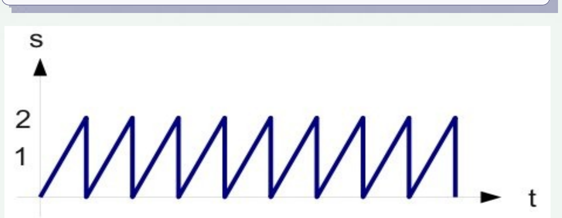
  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```

P(d,s,t) {d==0, s==0, t==0, 0<=n}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, **t<=n, s<=2,** 0<=t}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, t<=n, s<=2, 0<=t}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, t<=n, s<=2, 0<=t}



Bottom-up propagation of Transformers

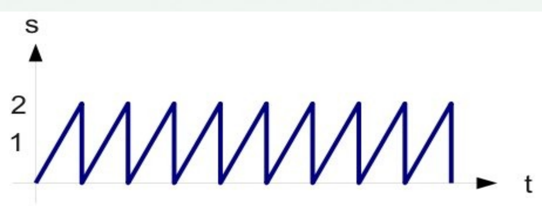
Car safety

Top-down propagation of Preconditions

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```



P(d,s,t) {d==0, s==0, t==0, 0<=n}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, **t<=n, s<=2,** 0<=t}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, t<=n, s<=2, 0<=t}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, t<=n, s<=2, 0<=t}

P(d,s,t) {0<=d, 5s<=9d, s+3<=3d+3t, d<=s+2t, s+3t<=3n+3, **t<=n+1, s<=3**, 3<=s+3t, 2<=s+2t}

Bottom-up propagation of Transformers

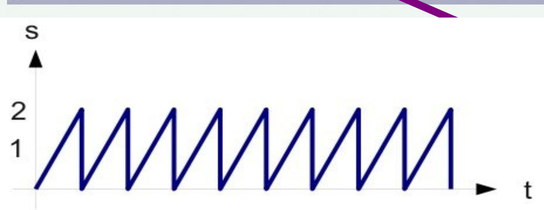
Car safety

Top-down propagation of Preconditions

```
void voiture05(int n)
{
  int s = 0, t = 0, d = 0;

  assert(n>=0);
  while(s <= 2 && t <= n) {
    if(alea() > 0.)
      t++, s = 0;
    else
      d++, s++;
  }

  if(d <= 2*n+3)
    printf("safe");
  else
    printf("crashed!");
}
```



P(d,s,t) {d==0, s==0, t==0, 0<=n}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, **t<=n,** **s<=2,** 0<=t}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, t<=n, s<=2, 0<=t}

P(d,s,t) {0<=d, s<=3d, s<=2d+2t, d<=s+2t, t<=n, s<=2, 0<=t}

P(d,s,t) {0<=d, 5s<=9d, s+3<=3d+3t, d<=s+2t, s+3t<=3n+3, **t<=n+1,** **s<=3**, 3<=s+3t, 2<=s+2t}

P() {0==-1}

Algorithm for Transitive Closure of Transformers

$\delta x^i = x^i - x^{i-1}$

Algorithm for Transitive Closure of Transformers

$\delta x^i = x^i - x^{i-1}$

$$x^k = x^0 + \sum_{i=1, k} \delta x^i$$

Algorithm for Transitive Closure of Transformers

II.2.1

$\delta x^i = x^i - x^{i-1}$

$$x^k = x^0 + \sum_{i=1,\, k} \delta x^i$$

$T(x^{i-1}, x^i)$

Algorithm for Transitive Closure of Transformers

II.2.1

$\delta x^i = x^i - x^{i-1}$

$$x^k = x^0 + \sum_{i=1, k} \delta x^i$$

$$T(x^{i-1}, x^i) \longrightarrow T'(\delta x^i) = \left\{ \delta x^i \mid \begin{array}{l} A\,\delta x^i = b \\ A'\,\delta x^i \leqslant b' \end{array} \right\}$$

Algorithm for Transitive Closure of Transformers

II.2.1

$\delta x^i = x^i - x^{i-1}$

$x^k = x^0 + \sum_{i=1,k} \delta x^i$ $\xrightarrow{\times A}$

$$A\,x^k = A\,x^0 + \sum_{i=1,k} A\,\delta x^i$$

$$A'\,x^k = A'\,x^0 + \sum_{i=1,k} A'\,\delta x^i$$

$T(x^{i-1}, x^i) \longrightarrow T'(\delta x^i) = \left\{ \delta x^i \mid \begin{array}{l} A\,\delta x^i = b \\ A'\,\delta x^i \leqslant b' \end{array} \right\}$

Algorithm for Transitive Closure of Transformers

II.2.1

$\delta x^i = x^i - x^{i-1}$

$x^k = x^0 + \sum_{i=1,k} \delta x^i$ —(× A)→

$A\, x^k = A\, x^0 + \sum_{i=1,k} A\, \delta x^i$

$A'\, x^k = A'\, x^0 + \sum_{i=1,k} A'\, \delta x^i$

$T(x^{i-1}, x^i) \rightarrow T'(\delta x^i) = \left\{ \delta x^i \mid A\, \delta x^i = b,\ A'\, \delta x^i \leqslant b' \right\}$ →

$A\, x^k = A\, x^0 + k\, b$

$A'\, x^k \leqslant A'\, x^0 + k\, b'$

Algorithm for Transitive Closure of Transformers

II.2.1

$\delta x^i = x^i - x^{i-1}$

$x^k = x^0 + \sum_{i=1,k} \delta x^i$ → $\times A$ →

$$A x^k = A x^0 + \sum_{i=1,k} A\,\delta x^i$$
$$A' x^k = A' x^0 + \sum_{i=1,k} A'\,\delta x^i$$

$T(x^{i-1}, x^i) \rightarrow T'(\delta x^i) = \left\{ \delta x^i \mid A\,\delta x^i = b,\ A'\,\delta x^i \leqslant b' \right\}$ →

$$A x^k = A x^0 + k\,b$$
$$A' x^k \leqslant A' x^0 + k\,b'$$

$$T^*(x^0, x) \subseteq \{ (x^0, x) \mid \exists k \in [0, \infty[\quad A x = A x^0 + kb \ \wedge\ A' x \leqslant A' x^0 + kb' \}$$

From Transformer T* to Precondition

II.3.1

- **Loop body Precondition from transformer T*:**

$$P^*(x^0) \subseteq \{ x \mid T^*(x^0,x) \}$$

- **Using T^+ instead of T*:**

$$P^* = P^0 \cup T^+(P^0)$$

- **Information loss due to convex hulls can be postponed even more:**

$$P^* = P^0 \cup T(P^0) \cup T^{2+}(P^0) \cup T(T^{2+}(P^0))$$

Improving Preconditions

Improving Preconditions

Periodic Behavior

Is it possible to parallelize Loop i ?

```
void flipflop(n,m)
{
  double x[2][m];
  int old = 0, new = 1, i, t = 0;

  while(t++<n) {
   for(i=0;i<m-1;i++)
     x[new][i] = g(x[old][i+1]);
   old = new;
   new = 1 - old;
  }
```

Periodic Behavior

Is it possible to parallelize Loop i ?

```
void flipflop(n,m)
{
  double x[2][m];
  int old = 0, new = 1, i, t = 0;

  while(t++<n) {
    for(i=0;i<m-1;i++)
      x[new][i] = g(x[old][i+1]);
    old = new;
    new = 1 - old;
  }
```

```
delete flipflop1

create flipflop1 flipflop1.c

setproperty
SEMANTICS_COMPUTE_TRANSFORMERS_IN_CONTEXT TRUE

setproperty SEMANTICS_FIX_POINT_OPERATOR "derivative"
setproperty PRETTYPRINT_ANALYSES_WITH_LF FALSE

echo TRANSFORMERS

activate PRINT_CODE_TRANSFORMERS
display PRINTED_FILE[flipflop]

echo PRECONDITIONS

activate PRINT_CODE_PRECONDITIONS
display PRINTED_FILE[flipflop]
```

Periodic Behavior


```
void flipflop(n,m)
{
  double x[2][m];
  int old = 0, new = 1, i, t = 0;

  while(t++<n) {
   for(i=0;i<m-1;i++)
     x[new][i] = g(x[old][i+1]);
   old = new;
   new = 1 - old;
  }
```

Is it possible to parallelize Loop i ?

```
void flipflop(int n, int m)
{
  double x[2][m];
  int old = 0, new = 1, i, t = 0;
// P(i,new,old,t) {new==1, old==0, t==0}

  while (t++<n) {
// P(i,new,old,t) {new+old==1, t<=n, 1<=t}
    for(i = 0; i <= m-2; i += 1)
// P(i,new,old,t) {new+old==1, 0<=i, i+2<=m, t<=n, 1<=t}
      x[new][i] = g(x[old][i +1]);
// P(i,new,old,t) {new+old==1, 0<=i, m<=i, t<=n, 1<=t}
    old = new;
// P(i,new,old,t) {new==old, 0<=i, m<=i, t<=n, 1<=t}
    new = 1-old;
// P(i,new,old,t) {new+old==1, 0<=i, m<=i, t<=n, 1<=t}
  }
```

Periodic Behavior

Is it possible to parallelize Loop i ?

```
void flipflop(n,m)
{
  double x[2][m];
  int old = 0, new = 1, i, t = 0;

  while(t++<n) {
   for(i=0;i<m-1;i++)
     x[new][i] = g(x[old][i+1]);
   old = new;
   new = 1 - old;
  }
```

Dependence test for x:
new == old
new == old+1
i' == i+1
$0 \leq i' \leq m - 2$
$0 \leq i \leq m - 2$
==> no solution

```
void flipflop(int n, int m)
{
  double x[2][m];
  int old = 0, new = 1, i, t = 0;
//  P(i,new,old,t) {new==1, old==0, t==0}

  while (t++<n) {
//  P(i,new,old,t) {new+old==1, t<=n, 1<=t}
     for(i = 0; i <= m-2; i += 1)
//  P(i,new,old,t) {new+old==1, 0<=i, i+2<=m, t<=n, 1<=t}
        x[new][i] = g(x[old][i +1]);
//  P(i,new,old,t) {new+old==1, 0<=i, m<=i, t<=n, 1<=t}
     old = new;
//  P(i,new,old,t) {new==old, 0<=i, m<=i, t<=n, 1<=t}
     new = 1-old;
//  P(i,new,old,t) {new+old==1, 0<=i, m<=i, t<=n, 1<=t}
  }
```

Virtual Unrolling

III.2.1

Virtual Unrolling


```
#include <stdio.h>
#include <assert.h>

void flipflop(n)
{
  int t, new,old;
  t=0,new=0,old=1;

  while (t<n) {
    new = 1 - new;
    old = 1 - old;
    t++;
  }
  assert(new+old==1);
}

int main(){
  int n;
  scanf("%d", &n);
  flipflop(n);
}
```

Virtual Unrolling


```
#include <stdio.h>
#include <assert.h>

void flipflop(n)
{
  int t, new,old;
  t=0,new=0,old=1;

  while (t<n) {
    new = 1 - new;
    old = 1 - old;
    t++;
  }
  assert(new+old==1);
}

int main(){
  int n;
  scanf("%d", &n);
  flipflop(n);
}
```

```
void flipflop(int n)
{
   int t, new, old;
//  P(new,old,t) {}
   t = 0, new = 0, old = 1;
//  P(new,old,t) {new==0, old==1, t==0}
   while (t<n) {
//  P(new,old,t) {t+1<=n, 0<=t}
      new = 1-new;
//  P(new,old,t) {1<=n, t+1<=n, 0<=t}
      old = 1-old;
//  P(new,old,t) {1<=n, t+1<=n, 0<=t}
      t++;
   }
//  P(new,old,t) {n<=t, 0<=t}
   new+old==1?(void) 0:__assert_fail("new+old==1", "./flipflop2.c",
14, (const char *) 0);
}
```

Virtual Unrolling

III.2.1

```
#include <stdio.h>
#include <assert.h>

void flipflop(n)
{
  int t, new,old;
  t=0,new=0,old=1;

  while (t<n) {
    new = 1 - new;
    old = 1 - old;
    t++;
  }
  assert(new+old==1);
}

int main(){
  int n;
  scanf("%d", &n);
  flipflop(n);
}
```

```
//  T() {}
void flipflop(int n)
{
//  T(new,old,t) {}
   int t, new, old;
//  T(new,old,t) {new==0, old==1, t==0}
   t = 0, new = 0, old = 1;
//  T(new,old,t) {new'==0, old'==1, t'==0, 1<=n, t+1<=n, 0<=t}
   while (t<n) {
//  T(new) {new+new'==1, 1<=n, t+1<=n}
      new = 1-new;
//  T(old) {old+old'==1, 1<=n, t+1<=n}
      old = 1-old;
//  T(t) {t==t'+1, 1<=n, t<=n}
      t++;
   }
}
```

Virtual Unrolling


```
#include <stdio.h>
#include <assert.h>

void flipflop(n)
{
  int t, new,old;
  t=0,new=0,old=1;

  while (t<n-1) {
    new = 1 - new;
    old = 1 - old;
    t++;
    new = 1 - new;
    old = 1 - old;
    t++;
  }
  if (t<n) {
    new = 1 - new;
    old = 1 - old;
    t++;
  }
    assert(new+old==1);
}
```

```
//  T() {}
void flipflop(int n)
{
//  T(new,old,t) {}
   int t, new, old;
//  T(new,old,t) {new==0, old==1, t==0}
   t = 0, new = 0, old = 1;
//  T(new,old,t) {new'==0, old'==1, t'==0, 1<=n, t+1<=n, 0<=t}
   while (t<n) {
//  T(new) {new+new'==1, 1<=n, t+1<=n}
      new = 1-new;
//  T(old) {old+old'==1, 1<=n, t+1<=n}
      old = 1-old;
//  T(t) {t==t'+1, 1<=n, t<=n}
      t++;
   }
}
```

Virtual Unrolling


```
#include <stdio.h>
#include <assert.h>

void flipflop(n)
{
 int t, new,old;
 t=0,new=0,old=1;

 while (t<n-1) {
  new = 1 - new;
  old = 1 - old;
  t++;
  new = 1 - new;
  old = 1 - old;
  t++;
 }
 if (t<n) {
  new = 1 - new;
  old = 1 - old;
  t++;
 }
  assert(new+old==1);
}
```

```
// P(new,old,t) {new==0, old==1, t==0}
  while (t<n-1) {
// P(new,old,t) {new==0, old==1, t+2<=n, 0<=t}
     new = 1-new;
// P(new,old,t) {new==1, old==1, 2<=n, t+2<=n, 0<=t}
     old = 1-old;
// P(new,old,t) {new==1, old==0, 2<=n, t+2<=n, 0<=t}
     t++;
// P(new,old,t) {new==1, old==0, 2<=n, t+1<=n, 1<=t}
     new = 1-new;
// P(new,old,t) {new==0, old==0, 2<=n, t+1<=n, 1<=t}
     old = 1-old;
// P(new,old,t) {new==0, old==1, 2<=n, t+1<=n, 1<=t}
     t++;
  }
// P(new,old,t) {new==0, old==1, n<=t+1, 0<=t}
  if (t<n) {
// P(new,old,t) {n==t+1, new==0, old==1, 1<=n}
     new = 1-new;
// P(new,old,t) {n==t+1, new==1, old==1, 1<=n}
     old = 1-old;
// P(new,old,t) {n==t+1, new==1, old==0, 1<=n}
     t++;  }
```

Virtual Unrolling


```
#include <stdio.h>
#include <assert.h>

void flipflop(n)
{
  int t, new,old;
  t=0,new=0,old=1;

  while (t<n-1) {
    new = 1 - new;
    old = 1 - old;
    t++;
    new = 1 - new;
    old = 1 - old;
    t++;
  }
  if (t<n) {
    new = 1 - new;
    old = 1 - old;
    t++;
  }
    assert(new+old==1);
}
```

```
delete flipflop2b
create flipflop2b flipflop2.c

setproperty
SEMANTICS_COMPUTE_TRANSFORMERS_IN_CONTEXT TRUE
setproperty PRETTYPRINT_ANALYSES_WITH_LF FALSE
setproperty SEMANTICS_FIX_POINT_OPERATOR "derivative"

setproperty SEMANTICS_K_FIX_POINT 2

echo TRANSFORMERS

activate PRINT_CODE_TRANSFORMERS
display PRINTED_FILE[flipflop]

echo PRECONDITIONS

activate PRINT_CODE_PRECONDITIONS
display PRINTED_FILE[flipflop]

close
quit
```

```
      t++;   }
```

Virtual Unrolling

III.2.1

```
#include <stdio.h>
#include <assert.h>

void flipflop(n)
{
  int t, new,old;
  t=0,new=0,old=1;

  while (t<n-1) {
    new = 1 - new;
    old = 1 - old;
    t++;
    new = 1 - new;
    old = 1 - old;
    t++;
  }
  if (t<n) {
    new = 1 - new;
    old = 1 - old;
    t++;
  }
    assert(new+old==1);
}
```

```
delete flipflop2b
create flipflop2b flipflop2.c

setproperty SEMANTICS_COMPUTE_TRANSFORMERS_IN_CONTEXT TRUE
setproperty PRETTYPRINT_ANALYSES_WITH_LF FALSE
setproperty SEMANTICS_FIX_POINT_OPERATOR "derivative"

setproperty SEMANTICS_K_FIX_POINT 2

echo TRANSFORMERS

activate PRINT_CODE_TRANSFORMERS
display PRINTED_FILE[flipflop]

echo PRECONDITIONS

activate PRINT_CODE_PRECONDITIONS
display PRINTED_FILE[flipflop]

close
quit
```

$$P^* = P^0 \cup T(P^0) \cup T^{2+}(P^0) \cup T(T^{2+}(P^0))$$

```
      t++;   }
```

Virtual Unrolling


```
#include <stdio.h>
#include <assert.h>

void flipflop(n)
{
  int t, new,old;
  t=0,new=0,old=1;

  while (t<n) {
    new = 1 - new;
    old = 1 - old;
    t++;
  }
  assert(new+old==1);
}

int main(){
  int n;
  scanf("%d", &n);
  flipflop(n);
}
```

```
delete flipflop2b
create flipflop2b flipflop2.c

setproperty
SEMANTICS_COMPUTE_TRANSFORMERS_IN_CONTEXT TRUE
setproperty PRETTYPRINT_ANALYSES_WITH_LF FALSE
setproperty SEMANTICS_FIX_POINT_OPERATOR "derivative"

setproperty SEMANTICS_K_FIX_POINT 2

echo TRANSFORMERS

activate PRINT_CODE_TRANSFORMERS
display PRINTED_FILE[flipflop]

echo PRECONDITIONS

activate PRINT_CODE_PRECONDITIONS
display PRINTED_FILE[flipflop]

close
quit
```

$$P^* = P^0 \cup T(P^0) \cup T^{2+}(P^0) \cup T(T^{2+}(P^0))$$

```
t++;   }
```

Virtual Unrolling


```
#include <stdio.h>
#include <assert.h>

void flipflop(n)
{
  int t, new,old;
  t=0,new=0,old=1;

  while (t<n) {
    new = 1 - new;
    old = 1 - old;
    t++;
  }
  assert(new+old==1);
}

int main(){
  int n;
  scanf("%d", &n);
  flipflop(n);
}
```

```
void flipflop(int n)
{
   int t, new, old;
//  P(new,old,t) {}
   t = 0, new = 0, old = 1;
//  P(new,old,t) {new==0, old==1, t==0}
   while (t<n) {
//  P(new,old,t) {new+old==1, t+1<=n, 0<=new, new<=1, new<=t}
      new = 1-new;
//  P(new,old,t) {new==old, 1<=n, t+1<=n, 0<=new, new<=1,
1<=new+t}
      old = 1-old;
//  P(new,old,t) {new+old==1, 1<=n, t+1<=n, 0<=new, new<=1,
1<=new+t}
      t++;
   }
}
```

```
      t++;   }
```

Iterative Refinment


```
int main()
{
  int i = 0, j = 0, n;
  if(n<0) exit(1);
  while(i<=n) {
    i++;
    j+=i;
  }
}
```

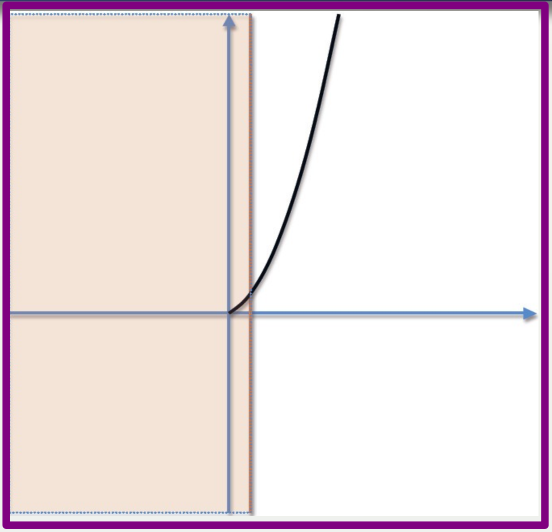
Iterative Refinment


```
int main()
{
  int i = 0, j = 0, n;
  if(n<0) exit(1);
  while(i<=n) {
    i++;
    j+=i;
  }
}
```

```
//  T(i,j) {i'==0, j'==0, 0<=i, i<=n, 0<=n}
   while (i<=n) {
//  T(i) {i==i'+1, i<=n+1, 0<=n}
      i++;
//  T(j) {i+j'==j, i<=n+1, 0<=n}
      j += i;
   }
//  T() {i==n+1, 1<=i}
```

Iterative Refinment


```
int main()
{
  int i = 0, j = 0, n;
  if(n<0) exit(1);
  while(i<=n) {
    i++;
    j+=i;
  }
}
```

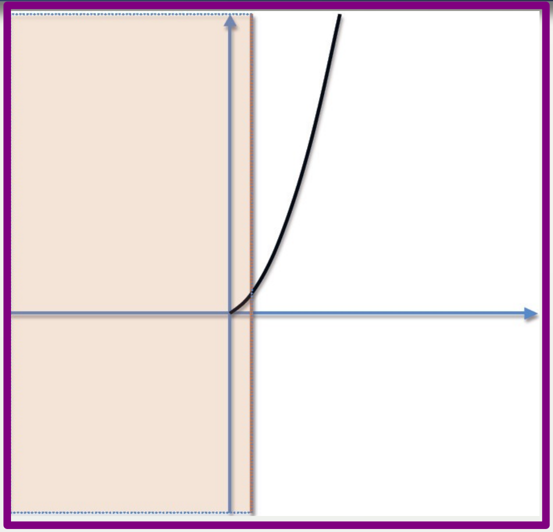


```
   int i = 0, j = 0, n;
//  P(i,j,n) {i==0, j==0}
   if (n<0)
//  P(i,j,n) {i==0, j==0, n+1<=0}
      exit(1);
//  P(i,j,n) {i==0, j==0, 0<=n}
   while (i<=n) {
//  P(i,j,n) {0<=i, i<=n}
      i++;
//  P(i,j,n) {1<=i, i<=n+1, 0<=n}
      j += i;
   }
//  P(i,j,n) {i==n+1, 1<=i}
```

```
//  T(i,j) {i'==0, j'==0, 0<=i, i<=n, 0<=n}
   while (i<=n) {
//  T(i) {i==i'+1, i<=n+1, 0<=n}
      i++;
//  T(j) {i+j'==j, i<=n+1, 0<=n}
      j += i;
   }
//  T() {i==n+1, 1<=i}
```

Iterative Refinment


```
int main()
{
  int i = 0, j = 0, n;
  if(n<0) exit(1);
  while(i<=n) {
    i++;
    j+=i;
  }
}
```

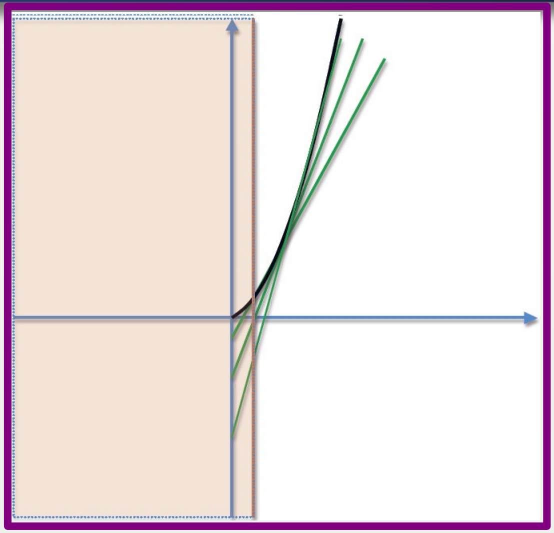


```
  int i = 0, j = 0, n;
//  P(i,j,n) {i==0, j==0}
  if (n<0)
//  P(i,j,n) {i==0, j==0, n+1<=0}
     exit(1);
//  P(i,j,n) {i==0, j==0, 0<=n}
  while (i<=n) {
//  P(i,j,n) {0<=i, i<=n}
     i++;
//  P(i,j,n) {1<=i, i<=n+1, 0<=n}
     j += i;
  }
//  P(i,j,n) {i==n+1, 1<=i}
```

```
//  T(i,j) {i'==0, j'==0, 0<=i, i<=j, 2i<=j+1, i<=n, 0<=n}
  while (i<=n) {
//  T(i) {i==i'+1, 1<=i, i<=n+1, 0<=n}
     i++;
//  T(j) {i+j'==j, 1<=i, i<=n+1, 0<=n}
     j += i;
  }
//  T() {i==n+1, 1<=i, 2i<=j+1, 3i<=j+3}
```

Iterative Refinment


```
int main()
{
  int i = 0, j = 0, n;
  if(n<0) exit(1);
  while(i<=n) {
    i++;
    j+=i;
  }
}
```



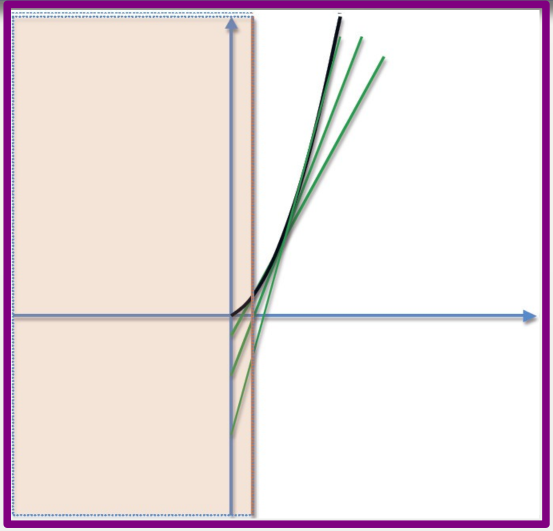
```
  int i = 0, j = 0, n;
// P(i,j,n) {i==0, j==0}
  if (n<0)
// P(i,j,n) {i==0, j==0, n+1<=0}
    exit(1);
// P(i,j,n) {i==0, j==0, 0<=n}
  while (i<=n) {
// P(i,j,n) {0<=i, i<=j, 2i<=j+1, 3i<=j+3, i<=n}
    i++;
// P(i,j,n) {1<=i, i<=j+1, 2i<=j+3, 3i<=j+6,
i<=n+1, 0<=n}
    j += i;
  }
// P(i,j,n) {i==n+1, 1<=i, 2i<=j+1, 3i<=j+3,
4i<=j+6}
```

```
// T(i,j) {i'==0, j'==0, 0<=i, i<=j, 2i<=j+1, i<=n, 0<=n}
  while (i<=n) {
// T(i) {i==i'+1, 1<=i, i<=n+1, 0<=n}
    i++;
// T(j) {i+j'==j, 1<=i, i<=n+1, 0<=n}
    j += i;
  }
// T() {i==n+1, 1<=i, 2i<=j+1, 3i<=j+3}
```

Iterative Refinment

III.3.1

```
int main()
{
  int i = 0, j = 0, n;
  if(n<0) exit(1);
  while(i<=n) {
    i++;
    j+=i;
  }
}
```



```
  int i = 0, j = 0, n;
// P(i,j,n) {i==0, j==0}
  if (n<0)
// P(i,j,n) {i==0, j==0, n+1<=0}
    exit(1);
// P(i,j,n) {i==0, j==0, 0<=n}
  while (i<=n) {
// P(i,j,n) {0<=i, i<=j, 2i<=j+1, 3i<=j+3, i<=n}
    i++;
// P(i,j,n) {1<=i, i<=j+1, 2i<=j+3, 3i<=j+6,
i<=n+1, 0<=n}
    j += i;
  }
// P(i,j,n) {i==n+1, 1<=i, 2i<=j+1, 3i<=j+3,
4i<=j+6}
```

```
// T(i,j) {i'==0, j'==0, 0<=i, i<=j, 2i<=j+1, 3i<=j+3,
//   4i<=j+6, i<=n, 0<=n}
  while (i<=n) {
// T(i) {i==i'+1, 1<=i, i<=j+1, 2i<=j+3, 3i<=j+6, i<=n+1, 0<=n}
    i++;
// T(j) {i+j'==j, 1<=i, i<=j'+1, 2i<=j'+3, 3i<=j'+6,
//   i<=n+1, 0<=n}
    j += i;
  }
// T() {i==n+1, 1<=i, 2i<=j+1, 3i<=j+3, 4i<=j+6, 5i<=j+10}
```

Iterative Refinment

III.3.1

```
int main()
{
  int i = 0, j = 0, n;
  if(n<0) exit(1);
  while(i<=n) {
    i++;
    j+=i;
  }
}
```

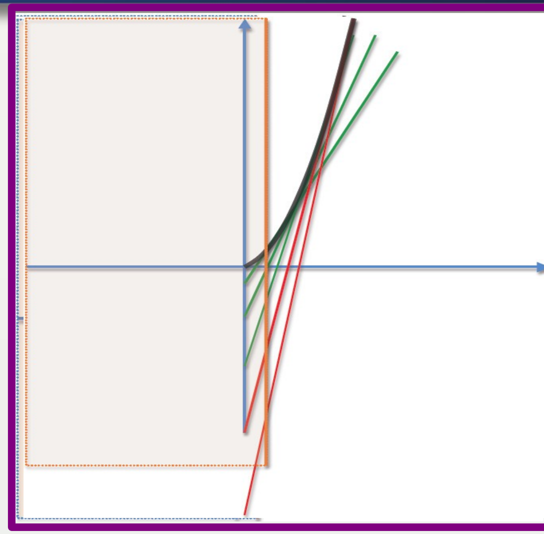
```
  int i = 0, j = 0, n;
//  P(i,j,n) {i==0, j==0}
  if (n<0)
//  P(i,j,n) {i==0, j==0, n+1<=0}
     exit(1);
//  P(i,j,n) {i==0, j==0, 0<=n}
  while (i<=n) {
//  P(i,j,n) {0<=i, i<=j, 2i<=j+1, 3i<=j+3,
4i<=j+6, 5i<=j+10, i<=n}
     i++;
//  P(i,j,n) {1<=i, i<=j+1, 2i<=j+3, 3i<=j+6,
4i<=j+10, 5i<=j+15,
//     i<=n+1, 0<=n}
     j += i;
  }
//  P(i,j,n) {i==n+1, 1<=i, 2i<=j+1, 3i<=j+3,
4i<=j+6, 5i<=j+10,6i<=j+15}
```

```
//  T(i,j) {i'==0, j'==0, 0<=i, i<=j, 2i<=j+1, 3i<=j+3,
//    4i<=j+6, i<=n, 0<=n}
   while (i<=n) {
//  T(i) {i==i'+1, 1<=i, i<=j+1, 2i<=j+3, 3i<=j+6, i<=n+1, 0<=n}
     i++;
//  T(j) {i+j'==j, 1<=i, i<=j'+1, 2i<=j'+3, 3i<=j'+6,
//    i<=n+1, 0<=n}
     j += i;
   }
//  T() {i==n+1, 1<=i, 2i<=j+1, 3i<=j+3, 4i<=j+6, 5i<=j+10}
```

Iterative Refinment

III.3.1

```
int main()
{
  int i = 0, j = 0, n;
  if(n<0) exit(1);
  while(i<=n) {
    i++;
    j+=i;
  }
}
```



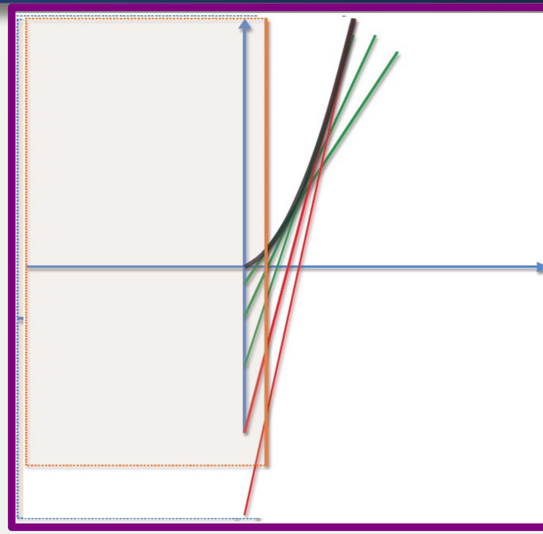
```
  int i = 0, j = 0, n;
// P(i,j,n) {i==0, j==0}
  if (n<0)
// P(i,j,n) {i==0, j==0, n+1<=0}
     exit(1);
// P(i,j,n) {i==0, j==0, 0<=n}
  while (i<=n) {
// P(i,j,n) {0<=i, i<=j, 2i<=j+1, 3i<=j+3,
4i<=j+6, 5i<=j+10, i<=n}
     i++;
// P(i,j,n) {1<=i, i<=j+1, 2i<=j+3, 3i<=j+6,
4i<=j+10, 5i<=j+15,
//   i<=n+1, 0<=n}
     j += i;
  }
// P(i,j,n) {i==n+1, 1<=i, 2i<=j+1, 3i<=j+3,
4i<=j+6, 5i<=j+10,6i<=j+15}
```

```
// T(i,j) {i'==0, j'==0, 0<=i, i<=j, 2i<=j+1, 3i<=j+3,
//   4i<=j+6, 5i<=j+10, 6i<=j+15, i<=n, 0<=n}
  while (i<=n) {
// T(i) {i==i'+1, 1<=i, i<=j+1, 2i<=j+3, 3i<=j+6, 4i<=j+10,
//   5i<=j+15, i<=n+1, 0<=n}
     i++;
// T(j) {i+j'==j, 1<=i, i<=j'+1, 2i<=j'+3, 3i<=j'+6,
//   4i<=j'+10, 5i<=j'+15, i<=n+1, 0<=n}
     j += i;
  }
// T() {..., 2i<=j+1, 3i<=j+3, 4i<=j+6, 5i<=j+10,
6i<=j+15,7i<=j+21}
```

Iterative Refinment

III.3.1

```
int main()
{
  int i = 0, j = 0, n;
  if(n<0) exit(1);
  while(i<=n) {
    i++;
    j+=i;
  }
}
```



```
  if (n<0)
// P(i,j,n) {i==0, j==0, n+1<=0}
     exit(1);
// P(i,j,n) {i==0, j==0, 0<=n}
  while (i<=n) {
// P(i,j,n) {0<=i, i<=j, 2i<=j+1, 3i<=j+3,
4i<=j+6, 5i<=j+10,
//    6i<=j+15, 7i<=j+21, i<=n}
     i++;
// P(i,j,n) {1<=i, i<=j+1, 2i<=j+3, 3i<=j+6,
4i<=j+10, 5i<=j+15,
//    6i<=j+21, 7i<=j+28, i<=n+1, 0<=n}
     j += i;
  }
// P(i,j,n) {i==n+1, 1<=i, 2i<=j+1, 3i<=j+3,
4i<=j+6, 5i<=j+10, 6i<=j+15, 7i<=j+21,
8i<=j+28}
```

```
// T(i,j) {i'==0, j'==0, 0<=i, i<=j, 2i<=j+1, 3i<=j+3,
//   4i<=j+6, 5i<=j+10, 6i<=j+15, i<=n, 0<=n}
  while (i<=n) {
// T(i) {i==i'+1, 1<=i, i<=j+1, 2i<=j+3, 3i<=j+6, 4i<=j+10,
//   5i<=j+15, i<=n+1, 0<=n}
     i++;
// T(j) {i+j'==j, 1<=i, i<=j'+1, 2i<=j'+3, 3i<=j'+6,
//   4i<=j'+10, 5i<=j'+15, i<=n+1, 0<=n}
     j += i;
  }
// T() {..., 2i<=j+1, 3i<=j+3, 4i<=j+6, 5i<=j+10,
6i<=j+15,7i<=j+21}
```

$$T^*_{n+1} = \tau(B, P^*_n) \wedge P^*_n$$

Control Restructuring


```
while {
  if(t) a;
  else b;
}
```

→

```
while(c) {
  while (c&&t) a;
  while (c&&!t) b;
}
```

Control Restructuring

Example – Gulwani 2007

```
#include <stdio.h>

int main()
{
  int x,y,z;

  x=0;
  y=50;
  while(x<100) {
   if (x<50)
     x++;
   else  {
     x++; y++;
   }
  }
  if (y==100)
   printf("property  verified\n");
}
```

```
#include <stdio.h>
int main() {
  int x,y,z;

  x=0;
  y=50;
  while(x<100)
   {
     while ( x<50)
       x++;
     while (x<100 && x>=50){
       x++; y++;
     }
   }
  if (x ==100 && y==100)
   printf("property  verified\n");
}
```

Control Restructuring

Example – Gulwani 2007

```
//  T(x,y) {x'==0, y'==50, x<=99, y<=x+50,
x+2450<=50y, 50<=y}
   while (x<100)
//  T(x,y) {x==x'+1, x+50y'<=50y+50, y'<=y, y<=y'+1}
      if (x<50)
//  T(x) {x==x'+1, x<=50}
         x++;
      else {
//  T(x,y) {x==x'+1, y==y'+1, 51<=x, x<=100}
         // BEGIN BLOCK
//  T(x) {x==x'+1, 51<=x, x<=100}
         x++;
//  T(y) {y==y'+1, 51<=x, x<=100}
         y++;
         // END BLOCK
      }
```

```
//  T() {x==0, y==50}
   while (x<100) {
//  T(x,y) {x==100, x'+y<=y'+100, y'+1<=y, y<=y'+50}
      // BEGIN BLOCK
//  T(x) {x<=49, x'<=x, x'<=49}
      while (x<50)
//  T(x) {x==x'+1, x<=50}
         x++;
//  T(x,y) {x+y'==x'+y, 50<=x, x<=99, 50<=x', x'<=99,
//     y'<=y}
      while (x<100&&x>=50) {
//  T(x,y) {x==x'+1, y==y'+1, 51<=x, x<=100}
         // BEGIN BLOCK
//  T(x) {x==x'+1, 51<=x, x<=100}
         x++;
//  T(y) {y==y'+1, 51<=x, x<=100}
         y++;
      }
```

Control Restructuring

Postponing convex hull

Example – Gulwani 2007

```
// T(x,y) {x'==0, y'==50, x<=99, y<=x+50,
x+2450<=50y, 50<=y}
  while (x<100)
// T(x,y) {x==x'+1, x+50y'<=50y+50, y'<=y, y<=y'+1}
    if (x<50)
// T(x) {x==x'+1, x<=50}
      x++;
    else {
// T(x,y) {x==x'+1, y==y'+1, 51<=x, x<=100}
      // BEGIN BLOCK
// T(x) {x==x'+1, 51<=x, x<=100}
      x++;
// T(y) {y==y'+1, 51<=x, x<=100}
      y++;
      // END BLOCK
    }
```

```
// T() {x==0, y==50}
  while (x<100) {
// T(x,y) {x==100, x'+y<=y'+100, y'+1<=y, y<=y'+50}
    // BEGIN BLOCK
// T(x) {x<=49, x'<=x, x'<=49}
    while (x<50)
// T(x) {x==x'+1, x<=50}
      x++;
// T(x,y) {x+y'==x'+y, 50<=x, x<=99, 50<=x', x'<=99,
//   y'<=y}
    while (x<100&&x>=50) {
// T(x,y) {x==x'+1, y==y'+1, 51<=x, x<=100}
      // BEGIN BLOCK
// T(x) {x==x'+1, 51<=x, x<=100}
      x++;
// T(y) {y==y'+1, 51<=x, x<=100}
      y++;
    }
```

Control Restructuring

Example – Gulwani 2007

```
// T(x,y) {x'==0, y'==50, x<=99, y<=x+50,
x+2450<=50y, 50<=y}
   while (x<100)
// T(x,y) {x==x'+1, x+50y'<=50y+50, y'<=y, y<=y'+1}
      if (x<50)
// T(x) {x==x'+1, x<=50}
         x++;
      else {
// T(x,y) {x==x'+1, y==y'+1, 51<=x, x<=100}
         // BEGIN BLOCK
// T(x) {x==x'+1, 51<=x, x<=100}
         x++;
// T(y) {y==y'+1, 51<=x, x<=100}
         y++;
         // END BLOCK
      }
```

```
// T() {
  while
// T(x,y
    // B
// T(x)
    wh
// T(x)
      x
// T(x,y
//   y'<
    wh
// T(x,y
      //
// T(x)
      x
// T(y)
      y
    }
```

```
// P(x,y,z) {x==0, y==50}
   while (x<100) {

// P(x,y,z) {x==0, y==50}
      while (x<50)
// P(x,y,z) {y==50, 0<=x, x<=49}
         X++;

// P(x,y,z) {x==50, y==50}
      while (x<100&&x>=50) {
// P(x,y,z) {x==y, 50<=x, x<=99}
         x++;
// P(x,y,z) {x==y+1, 51<=x, x<=100}
         y++;
      }
   }

// P(x,y,z) {x==100, x==y}
   if (y==100)
      printf("property  verified\n");}
```

Control Restructuring

III.4.1

Example – Gulwani 2007

```
// P(x,y,z) {x==0, y==50}
   while (x<100)

// P(x,y,z) {x<=99, y<=x+50, x+1200<=25y, 50<=y}
      if (x<50)
// P(x,y,z) {x<=49, y<=x+50, x+1200<=25y, 50<=y}
         x++;
      else {
// P(x,y,z) {50<=x, x<=99, y<=x+50, x+1200<=25y,50<=y}
         x++;
//P(x,y,z) {51<=x, x<=100, y<=x+49, x+1199<=25y, 50<=y}
         y++;
      }

// P(x,y,z) {x==100, 53<=y, y<=150}
   if (y==100)
      printf("property  verified\n");
}
```

```
// P(x,y,z) {x==0, y==50}
   while (x<100) {

// P(x,y,z) {x==0, y==50}
      while (x<50)
// P(x,y,z) {y==50, 0<=x, x<=49}
         X++;

// P(x,y,z) {x==50, y==50}
      while (x<100&&x>=50) {
// P(x,y,z) {x==y, 50<=x, x<=99}
         x++;
// P(x,y,z) {x==y+1, 51<=x, x<=100}
         y++;
      }
   }

// P(x,y,z) {x==100, x==y}
   if (y==100)
      printf("property  verified\n");}
```

Related Work

IV.0.1

- Many examples from Chaochen, Gonnord, Gopan, Gulavani, Gulwani, Halbwachs, Merchat,... processed successfully
- Gonnord: abstract acceleration
- Kelly & al.: transitive closure computation of relation encoded by a Presburger formulae. Heuristic uses the *d-form* relation
- Bielecki & al.: exact non linear transitive closure for normalized relations, written as systems of recurrence equations
- Paige & Koenig: finite differencing of source code
  - *Extension to predicates over arrays ?*
- Spezialetti & Gupta: monotonicity analysis

Conclusion

IV.1.1

- **New transitive closure algorithm, implemented in PIPS**
- **Input : programs in Fortran and C**
  - Large scientific codes up to hundreds of functions, 100KLOCS
  - Bourdoncle's algorithm used to deal with unstructured control flow graph
    - But not optimal
  - Structured loops converted into while loops
- **Modular and interprocedural Analysis**
- **Results are equivalent to related work examples**
  - Halbwachs extended with symbolic time bound
  - Beyond counters: multiply, divide, affine assignments
- **Extensions for non affine program behavior:**
  - Periodic
  - Iterative transformer refinement using preconditions